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Initial Value Problems in Quaternionic Analysis

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Abstract

Recently the initial value problem

$$\partial_t u = \mathcal{L}u := \sum_{i=1}^3 A^{(i)}(t,x) \partial_{x_i} u + B(t,x)u + C(t,x)$$
$$u(0,x) = u_0(x)$$

has been solved uniquely by N. Q. Hung [1] using the method of *associated spaces* constructed by W. Tutschke [2] in the space of generalized regular functions in the sense of quaternionic analysis satisfying the equation

$$\mathcal{D}_{\alpha}u := \mathcal{D}u + \alpha u = 0, \quad \alpha \in \mathbb{R}$$

where $\mathcal{D} = \sum_{j=1}^{3} e_j \partial_{x_j}$ is the DIRAC operator, and t is the time variable. Only sufficient conditions has been obtained in [1] for the operators \mathcal{L} and \mathcal{D}_{α} to be associated.

In the present talk we will prove necessary and sufficient conditions for the underlined operators to be associated. This criterion makes it possible to construct all linear operators \mathcal{L} for which the initial value problem with an arbitrary initial generalized regular function is always solvable. Further we will correct a mistake made in the calculation of the interior estimate in [1].

Key Words: Initial value problems, associated operators, quaternionic analysis, Dirac operator.

References

- N. Q. Hung, Initial Value Problems in Quaternionic Analysis with a Disturbed Dirac Operator, Adv. appl. Clifford alg., Vol. 22, Issue 4 (2012), pp. 1061-1068.
- [2] W. Tutschke, Solution of initial value problems in classes of generalized analytic functions, Teubner Leipzig and Springer Verlag, 1989.