

MATE 316

Spring 2020

Homework # 2

Due March 13th, 2020 (lecture time)

Group submission (up to 3 students per group) is allowed.

1

Question 1:

Estimate the number of crystal-like spherical clusters containing a) 14 atoms and b) 140 atoms in 1 mm³ of liquid copper at 1306 K.

The atomic volume of liquid copper (Ω) is 1.6×10^{-29} m³,

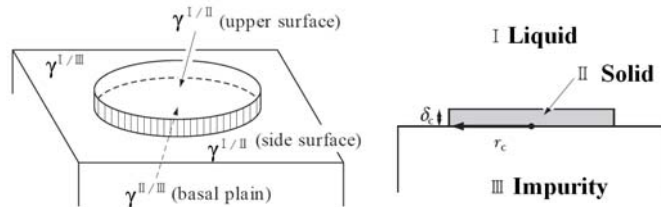
solid-liquid interface energy (γ_{SL}) is 0.177 J/m²,

$k = 1.38 \times 10^{-23}$ J/K, $T_m = 1356$ K.

The enthalpy of melting of Cu is -13.05 kJ/mole.

2

Question 2: Consider the heterogeneous disk-like nucleation of pure metal M from liquid on an impurity surface as shown below.



a) Write down the expression for the total change in free energy due to nucleation if the disk has a radius of r and a thickness of δ .

b) Determine the critical values of r (r_c) and δ (δ_c) based on the following information:

$$r = 15\delta \quad \gamma^{I/II} = 0.15 \text{ J/m}^2 \quad \gamma^{II/III} = 0.09 \text{ J/m}^2 \quad \gamma^{I/III} = 0.23 \text{ J/m}^2$$

$$G_{\text{liquid}} = -160 \text{ J/mole}$$

$$G_{\text{solid}} = -200 \text{ J/mole}$$

$$\text{Density of M} = 3 \text{ g/cm}^3$$

$$\text{Molecular weight of M} = 30 \text{ g/mole}$$

Question 3:

For pure metals, the fraction of material to undergo dendritic solidification (f_{dend}) can be calculated by the following formula:

$$f_{\text{dend}} = \frac{c^* \Delta T}{\Delta H_f}$$

Where c is the specific heat of the liquid, ΔT is the amount of undercooling and ΔH_f is the latent heat of solidification.

Calculate the fraction of dendritic solidification for pure gold if the nucleation starts at

- 10°C of undercooling;
- 100°C of undercooling; and
- homogeneously.

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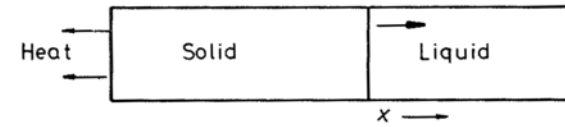
Question 4:

Estimate the temperature gradient to be maintained within solid aluminum so that the planar solidification front moves into liquid aluminum maintained at its melting point at a velocity of 1.5 mm/s.

For Al: Thermal conductivity = 225 W/mK, latent heat of fusion = 398 kJ/kg, density = 2700 kg/m³, $T_{\text{melting}} = 660.4^{\circ}\text{C}$.

Question 5:

Consider the unidirectional solidification of an alloy with composition X_0 .



Sketch how the temperature of the solid/liquid interface varies as the solidification proceeds from 0 to 100% when the solidification is under the conditions of

- equilibrium conditions,
- no diffusion in solid and perfect mixing in liquid, and
- no diffusion in solid and only diffusional mixing in liquid.

Use the phase diagram given and indicate the exact temperatures when possible.

Question 5:

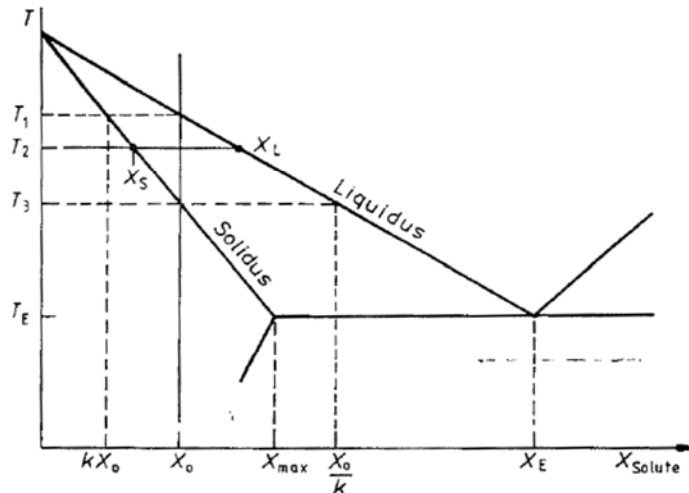


Fig. 4.19 A hypothetical phase diagram. $k = X_S/X_L$ is constant.

Question 6:

The variation of the solute concentration along a bar after a single pass zone melting (left to right) is shown below (not scaled). Following this operation, the bar is single pass zone melted in the reverse direction (right to left). Draw the final variation of solute concentration along the bar.

